Dynamic Systems and Applications 29 (2020) 87-100

IDENTIFICATION OF STRUCTURAL DAMAGE TO TRANSMISSION TOWER STRUCTURE BASED ON DYNAMIC FINGERPRINTING AND BAYES DATA FUSION

XIUFENG WU[1,*], YONGFENG ZUO[1], LIANG BING[1], CHUNLING YANG[1], AND ZHENGNAN GE[2]

[1]LiaoNing Technical University, FuXin City, LiaoNing Province, China
[2]YuRun Group, NanJing City, JiangSu Province, China
*Corresponding author. E-mail: wuxiufeng@lntu.edu.cn

ABSTRACT. This paper focuses on damage identification in transmission tower structure during operation. Using the dynamic principle of fingerprint identification, studied that the structural damage identification effect of two characteristic factors, curvature modal loss and average numerical modal strain energy of transmission tower structure. Then, Bayes data fusion is carried out with these two kinds of data, and the probability model of transmission tower structure damage calculation based on Bayes network classifier is established. Then, the study analyzes the impact of noise on the accuracy of damage identification. The results show that Bayes data fusion based on curvature modal loss and average numerical modal strain energy is beneficial to the identification of transmission tower structure damage location and degree, and improves the identification calculation efficiency and accuracy.

AMS Subject Classification. 15B15

Keywords. dynamic fingerprint; curvature modal loss; modal strain energy; data fusion; Bayes network; structural damage identification.

1. Introduction

Transmission tower structure (TTS) is complex as it encompasses features such as high structure and long span (Fig. 1); entails use of a spatial lattice; is easily affected by environmental loads such as wind, rain, and earthquakes; and experiences tower line-coupled vibrations [1]. Given their long-term service periods, these structures are bound to endure structural damage, which cumulates over time, making them prone to collapse under extreme conditions. Thus, long-term monitoring of the health of transmission tower structures in crucial to ensure the safety and continued operation of the power grid.

Received December 20, 2019

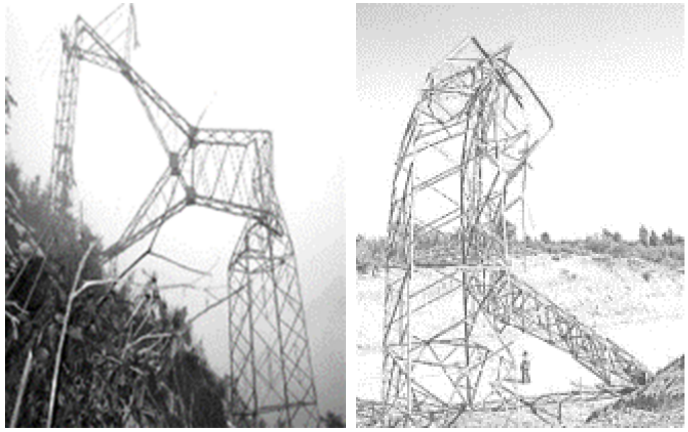


Figure 1. Seismic damage map of transmission tower

In recent decades, research on structural damage identification has been a popular topic in the engineering field and remarkable progress has been made in this area. The basic power fingerprints studied thus far can be generalized into four categories [2]: direct modal parameter class (encompassing parameters such as frequency and vibration mode) [3], modal parameter function class (including parameters such as the square of frequency, modal strain energy, modal guarantee criteria, coordinate modality assurance criteria, modal flexibility, and residual force vector) [4], curvature class (including parameters such as modal curvature, and flexibility curvature) [5], and vibration signal non-modal processing (encompassing Fourier transform, short-time Fourier transform, wavelet/package analysis, blind source separation, and so on. Generally, using strain parameters (strain, modal strain energy, curvature mode, etc.) as the basis for identifying damage location is preferable to utilizing displacement parameters (displacement, displacement mode, and flexibility matrix).

In recent years, the data fusion method has been applied to structural damage identification. This method avoids the misjudgment in the degree of damage that using a single index (modal curvature and modal strain energy) would entail, and confers the advantage of strong noise immunity as it modifies and merges multi-sensor data. Currently, popular data fusion methods include neural networks, Bayes theory [6], D-S evidence theory [7], and Fuzzy set theory [8]. Bayes data fusion method, which is effective when there is less data, can deal with multiple types of problems.

This study divides the discussion on damage identification into two parts. First, it is imperative to locate the damage and estimate the extent of damage to the TTS using the finite element model (FEM). Two damage identification methods, namely, the mean curvature mode difference factor and the average modal strain energy change rate, are used in this study. Second, fusing the dynamic fingerprint data through Bayes reasoning allows accurate location identification of difficult to detect structural damage in the presence of noise. Third, the Bayes network classification algorithm is applied to analyze different damage conditions. The proposed numerical simulation method provides ideal test results, wherein even minor damage can be detected easily. The validity of this method was thus verified.

2. Evaluation indexes for damage location in TTS

Curvature-based damage identification was proposed by Pandey and Biswas (1994) [9]. This method has been shown to have good sensitivity to structural damage. Stubbs et al. were the first to devise a formula for modal strain energy before and after damage occurred using the Euler-Bernoulli beam theory [10]. Thus, they deduced the damage index related to the stiffness before and after damage. Thereafter, many scholars have helped improve upon the identification of damage location in complex structures. Sohn and Law(2015) extended Bayes model correction method to the case of multiple damage of structures, and solved the multiple damage identification problems of multi-storey frame structures and concrete bridge piers respectively [11]. The Bayes model correction method for on-line monitoring of the structure, use a series of identified modal parameters are calculated by continuous correction model stiffness parameters, in a specific ratio is less than the corresponding initial model of stiffness parameters of probability, the maximum possible reduction of stiffness at a certain position within the structure is taken as a pointer to damage at the corresponding position.

2.1. Average curvature mode damage factor ($\mathbf{CMD}_{ave}$).

According to material mechanics, the relationship between curvature and bending stiffness can be expressed as

$$k(x) = \frac{1}{\rho(x)} = \frac{M(x)}{E(x)I(x)} \tag{1}$$

where $k(x)$ is the curvature, $\rho(x)$ is the radius of curvature after deformation of the beam axis, $M(x)$ is the bending moment, and $E(x)I(x)$ is the bending stiffness. x refers to the beam's axis coordinate.

Curvature modal analysis cannot provide direct estimates; it provides an approximate calculation using the displacement mode provided by the formula on curvature center difference. Considering order i and modal node j, curvature k_{ij} is given by Yang et al. (2015) [12].

$$\mathrm{k}_{ij} = \frac{\emptyset_{i(j-1)} - 2\emptyset_{ij} + \emptyset_{i(j+1)}}{(\Delta x)^2} \tag{2}$$

$\emptyset_{i(j-1)}, \emptyset_{ij}$, and $\emptyset_{i(j+1)}$ denote the displacement mode values for order i and modal nodes $j-1, j$ and $j+1$, respectively, and Δx is the distance between two adjacent points.

Considering order i and modal node j, the curvature mode difference factor CMD_{ij} is defined as

$$\mathrm{CMD}_{ij} = \left|k_{ij}^d - k_{ij}^u\right| \tag{3}$$

where k_{ij}^u and k_{ij}^d respectively are the curvature of order i and modal node j before and after the structural damage occurs.

For multiple orders i and node j, the following formula for average curvature mode damage factor is applicable.

$$\mathrm{CMD}_{avej} = \frac{1}{m}\sum_{i=1}^{m} CMD_{ij} \tag{4}$$

Here, m is the total number of modes.

2.2. **Average modal strain energy change rate (MSE_{ave}).** The modal strain energy MSE_{ij}^u and MSE_{ij}^d before and after structural damage for order i and modal node j can be respectively defined as [13]:

$$\mathrm{MSE}_{ij}^u = \frac{1}{2}\left\{\emptyset_{i,j}^u\right\}^T\left[K_j^u\right]\left\{\emptyset_{i,j}^u\right\} \tag{5}$$

$$\mathrm{MSE}_{ij}^d = \frac{1}{2}\left\{\emptyset_{i,j}^d\right\}^T\left[K_j^d\right]\left\{\emptyset_{i,j}^d\right\} \tag{6}$$

where $\left\{\emptyset_{i,j}^u\right\}$ and $\left\{\emptyset_{i,j}^d\right\}$ respectively denote the vibration mode vector before and after structural damage, and $\left[K_j^u\right]$ and $\left[K_j^d\right]$ respectively refer to the element stiffness matrix before and after structural damage. When element j is damaged and its struffness is unknown, one can use element j' stiffness matrix in the calculation instead.

An elements modal strain energy variety (MSEC) before and after structural damage can be expressed as below (note that higher orders are disregarded here).

$$\mathrm{MSE}_{i,j} = \mathrm{MSE}_{i,j}^d - \mathrm{MSE}_{i,j}^u \tag{7}$$

Element MSEC is a factor sensitive to the location of the structural damage. Thus, it can be used to judge the position of damage in a structure (Wei et al. 2016) [14]. This indicator, also called the modal strain energy rate (MSE), can be expressed as

$$\mathrm{MSE}_{i,j} = \frac{\left|MSE_{i,j}^d - MSE_{i,j}^u\right|}{MSE_{i,j}^u} \tag{8}$$

To reduce the influence of noise in the modal vibration mode, one uses the multimodal mode vibration mode to judge the positions of damage in the structure. The average modal strain energy change rate AMSECR_j for element j and modal vibration mode m is defined as

$$\mathrm{MSE}_{avej} = \frac{1}{m}\sum_{i=1}^{m} MSE_{i,j} \tag{9}$$

Before calculating the structures strain energy, the minimal error method can be extended with the tested vibration mode to an untested degree of freedom [15].

Dividing the vibration mode into two parts is dependent on whether the test has been completed. Thus, the ith order vibration mode equation can be written as follows:

$$\left\{\begin{bmatrix} [K_{\alpha\alpha}] & [K_{\alpha\beta}] \\ [K_{\beta\alpha}] & [K_{\beta\beta}] \end{bmatrix} - \tau_i \begin{bmatrix} [M_{\alpha\alpha}] & [M_{\alpha\beta}] \\ [M_{\beta\alpha}] & [M_{\beta\beta}] \end{bmatrix}\right\}\left\{\begin{matrix} \{\emptyset_{\alpha i}\} \\ \{\phi_{\beta i}\} \end{matrix}\right\} \approx \{0\} \tag{10}$$

where α and β respectively correspond to the measured and untested degrees of freedom. In order to facilitate analysis, the matrix in the left-hand bracket of the equation is denoted as $[N]$, such that

$$[N] = \begin{bmatrix} [K_{\alpha\alpha}] & [K_{\alpha\beta}] \\ [K_{\beta\alpha}] & [K_{\beta\beta}] \end{bmatrix} - \tau_i \begin{bmatrix} [M_{\alpha\alpha}] & [M_{\alpha\beta}] \\ [M_{\beta\alpha}] & [M_{\beta\beta}] \end{bmatrix} = \begin{bmatrix} [N_{\alpha\alpha}] & [N_{\alpha\beta}] \\ [N_{\beta\alpha}] & [N_{\beta\beta}] \end{bmatrix} \tag{11}$$

After extending the order, the ith order modal's residual vector $\{\delta_i\}$ is expressed as seen below and the formula's right-hand side becomes $\{0\}$.

$$\{\delta_i\} = \begin{bmatrix} [N_{\alpha\alpha}] \\ [N_{\beta\alpha}] \end{bmatrix}\begin{bmatrix} [N_{\alpha\beta}] \\ [N_{\beta\beta}] \end{bmatrix}\left\{\left\{\begin{matrix} \{\emptyset_{\alpha i}\} \\ \{\{\phi_{\beta i}\} \end{matrix}\right\}\right. \tag{12}$$

The square sum error of mode i is expressed as S_i

$$S_i = \{\delta_i\}^T\{\delta_i\} = \left[\{\emptyset_{\alpha i}\}^T\{\emptyset_{\beta i}\}^T\right]$$

$$\begin{bmatrix} [N_{\alpha\alpha}]^T & [N_{\beta\alpha}]^T \\ [N_{\beta\alpha}]^T & [N_{\beta\beta}]^T \end{bmatrix}\begin{bmatrix} [N_{\alpha\alpha}] & [N_{\alpha\beta}] \\ [N_{\beta\alpha}] & [N_{\beta\beta}] \end{bmatrix}\left\{\begin{matrix} \{\emptyset_{\alpha i}\} \\ \{\phi_{\beta i}\} \end{matrix}\right\} \tag{13}$$

If $\frac{\mu S_i}{\mu\{\rho_{\beta i}\}} = \{0\}$, S_i attains its minimum value, such that

$$\begin{aligned}\{\emptyset_{\beta i}\} = &-\{[N_{\beta\alpha}][N_{\alpha\alpha}] + [N_{\beta\beta}][N_{\beta\alpha}]\}\{\emptyset_{\alpha i}\}\{[N_{\beta\alpha}][N_{\alpha\beta}] \\ &+ [N_{\beta\beta}][N_{\beta\beta}]\}^{-1}\end{aligned} \tag{14}$$

3. **Damage identification based on Bayes theorem data fusion**

3.1. **Bayes theorem data fusion method.** The application of Bayes theorem in data fusion can be expressed such that $A = A_1, A_2, \ldots, A_n$ for n recognition targets,

$B = \{B_1, B_2, \ldots, B_m\}$ for sensor m, and prior probability is $P(A_i)$. The following conditional probability matrix can be obtained for sensor m:

$$\begin{bmatrix} P(B_1|A_1) & \cdots & P(B_1|A_n) \\ \vdots & \ddots & \vdots \\ P(B_m|A_1) & \cdots & P(B_m|A_n) \end{bmatrix}$$

The recognition probability of target A_i is

$$\begin{aligned} P(A_i|B) &= \frac{P(B|A_i)P(A_i)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)} \\ &= \frac{\prod_{j=1}^{m} P(B_j|A_i)P(A_i)}{\sum_{i=1}^{n}\left[\prod_{j=1}^{n} P(B_j|A_i)P(A_i)\right]} \end{aligned} \tag{15}$$

3.2. **Application of Bayes classifier.** Bayes classifier uses the Bayes network classification algorithm to identify the degree of structural damage. This network includes class node C, where the value of C is derived from the class collection $(C_1, C_2, \ldots, C_n)$, and it also contains a set of nodes $X = (x_1, x_2, \ldots, x_m)$, which are features for the classification. To enable TTS damage identification using the Bayes network classification algorithm, a damage location is identified alongside the classification characteristic value for $X = (x_1, x_2, \ldots, x_m)$. The probability of damage degree for category C_i is denoted as:

$$\mathrm{P}(C = C_i|X = \mathrm{x}) = \frac{P(X = \mathrm{x}|C = C_i)P(C = C_i)}{P(X = x)} \tag{16}$$

The Bayes network classifier can be categorized into two stages. The first stage is Bayes network classifier learning, namely, constructing classifiers from sample data, as well structure learning and conditional probability tables (CPTs). This stage analyzes dynamic fingerprint data sets to arrive at a characteristic value, and calculates the damage degree as a set of classes. In the second stage, the results of the structure learning and CPTs aid the completion of Bayes classifier network reasoning; the conditional probability of class nodes is calculated and the test sample is classified. The process for TTS damage identification is shown in Fig. 2.

4. Numerical simulation of damage identification based on Bayes data fusion

4.1. **Damage model for TTS.** This study referred to cup-type TTS and conducted the numerical simulation using the lumped mass point method to attach the wires and insulators to the suspension point. The height of the TTS was assumed to be 51 m. ANSYS software was used to model the beam and truss elements, that structural steel is used for all the truss members. The basic parameters of the FEM were as follows: elastic modulus E = 2.06×10^5MPa and density = 7800kg/m^3. The rod elements

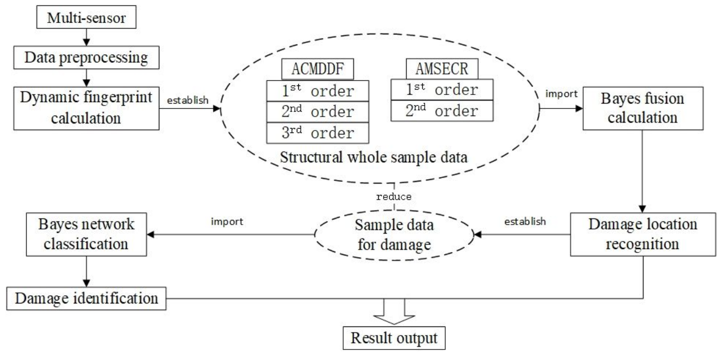
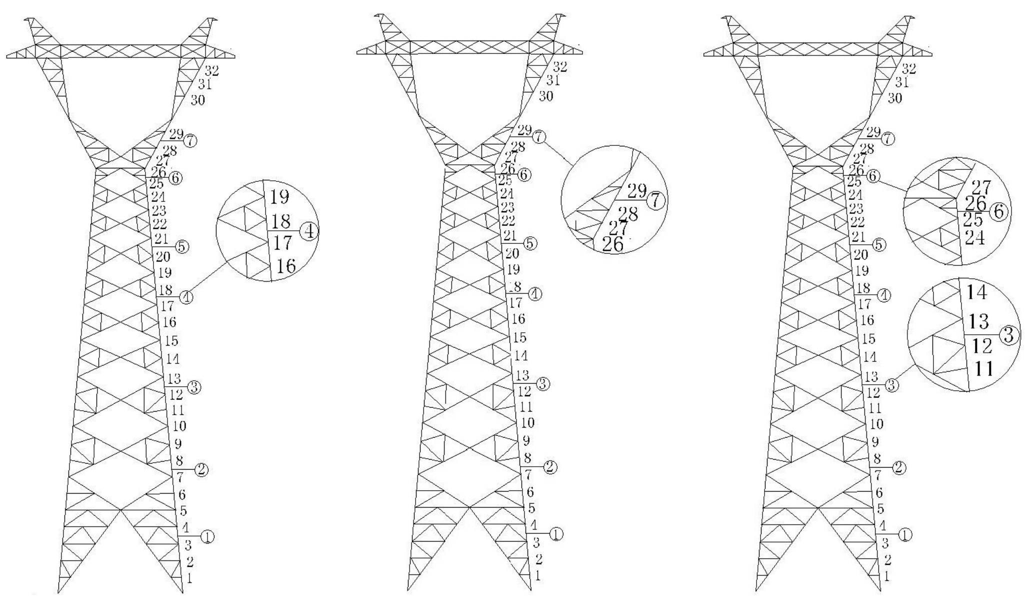


FIGURE 2. Flow chart for structural damage identification

were modeled using LINK8 element and the unit node was articulated. The beams were modeled using BEAM188. Fig. 3 shows the elevation layout of the TTS, where the plain numbers are the node numbers, and the number in the circle expresses the number of damaged bars. The method used to identify damage to the internal rods was the same as that for the external support rods, and the degree of damage was estimated in terms of the elastic modulus of the rod. The various damage conditions pertaining to the TTS are shown in table 1.



(a) Working condition 1 (b) Working condition 2 (c) Working condition 3

FIGURE 3. Layout of TTS facade and sensor measurement points

Because the seismic damage of the cup-type TTS often occurs at the neck of the tower and below, this is the basis for the arrangement of working conditions in this paper. The main character of component failure is the buckling of the member, so the damage degree is realized by reducing the elastic modulus of the member.

Table 1. Working and damage conditions of TTS

Working condition	Damage type	Damage unit	Damage degree (%)
W1 (Fig. 3-a)	Single location	No.4	5
W2 (Fig. 3-b)	Single location	No.7	10
W3 (Fig. 3-c)	Multiple locations	No.3	10
		No.6	5

4.2. **Noise addition and data preprocessing.** In a realistic engineering environment, the frequency index of the structure is usually measured accurately, but the mode index is easily influenced by environmental excitation of the test environment. In this paper, the environmental noise measured by the device was assumed to be white noise conforming to the Gaussian distribution. According to Kiviniemi and Sauerwein (2016), noise may be added for the mass normalized mode as seen below [16].

$$\widetilde{\emptyset}_{j,i} = \emptyset_{j,i}\left(1 + \frac{\delta_i p \emptyset_{\max,i}}{100}\right) \tag{17}$$

where $\emptyset$ and $\widetilde{\emptyset}$ respectively are the first component j of mode i before and after the addition of noise, δ_i is the Gauss random number with a mean of 0 and a variance of $1, \emptyset_{\max,i}$ is the maximum component for the absolute value of mode i, and p represents the noise level (which is the same as the level of Gauss white noise based on the measured modal shape). Data preprocessing was used to reduce the influence of noise. This study calculated the arithmetic mean value for 10 sets of noise.

4.3. **Identification of structural damage locations.** Assuming the noise level is 5%, this study calculated the position of each of the first three-order CMD_{aves} for all three working conditions listed in table 1 using formula (1). The first two-order MSE aves were calculated using formula (2).

The corresponding results for TTS damage location identification are shown in Fig. 4-5 (for W 1), Fig. 7-8 (for W 2), and Fig. 10-11 (for W 3). The study considered the above-mentioned five groups (first three-order $CMD_{\text{aves and}}$ first two-order MSE_{aves}) under all working conditions for the dynamic fingerprint data fusion Bayes ian network measurement. Suppose each unit $\mathrm{E}_i (i = 1, 2, \ldots, 10)$ has equal prior probability, $\mathrm{P}(\mathrm{E}_i) = 1/10 (i = 1, 2, \ldots, 10)$. The results of structural damage identification after the fusion of the Bayes formula are shown in Fig. 6 (W 1), Fig. 9 (W 2), and Fig. 12 (W 3).

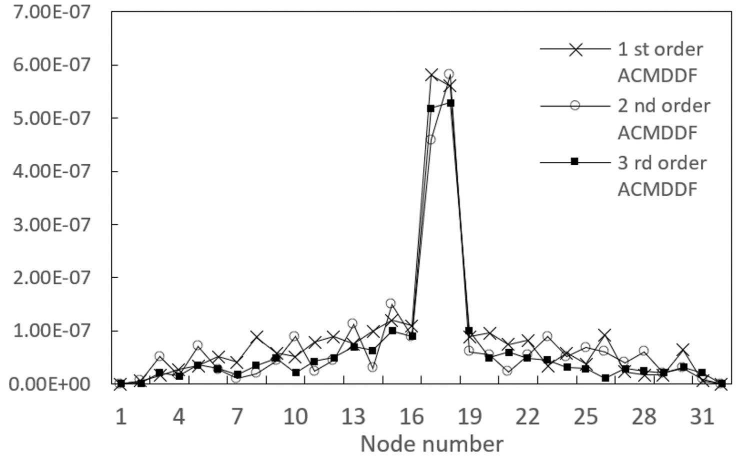


Figure 4. Identification results by CMD_{ave}(W1)

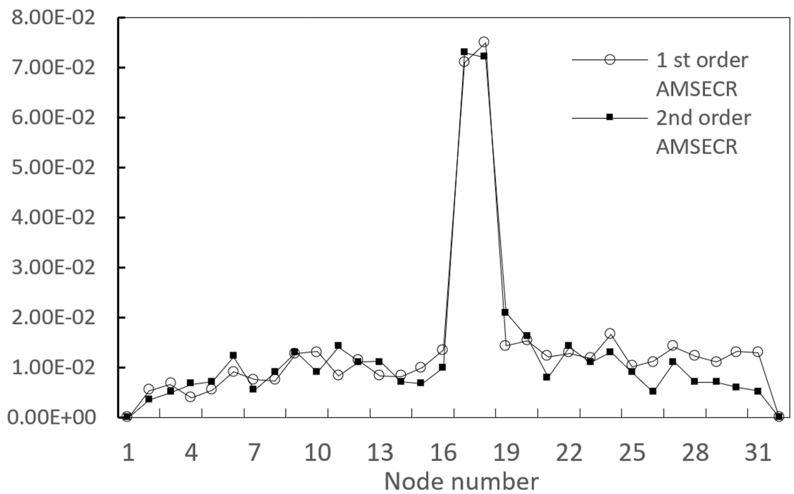


Figure 5. Identification results by MSE_{ave}(W1)

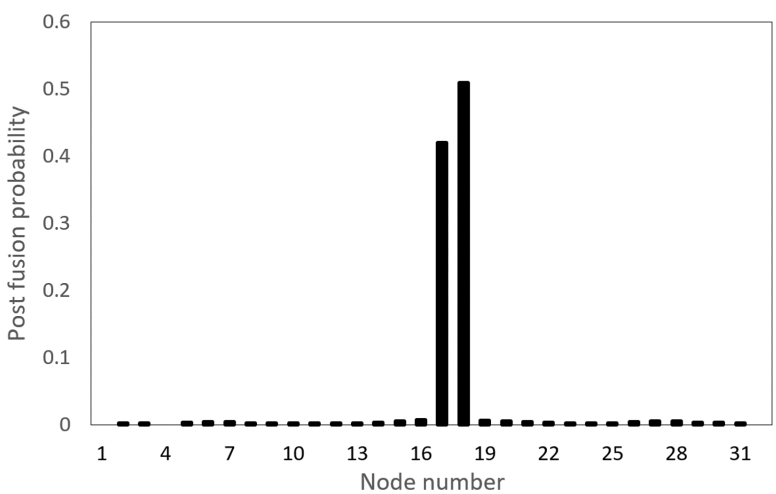


Figure 6. Results by the Bayes fusion method (W1)

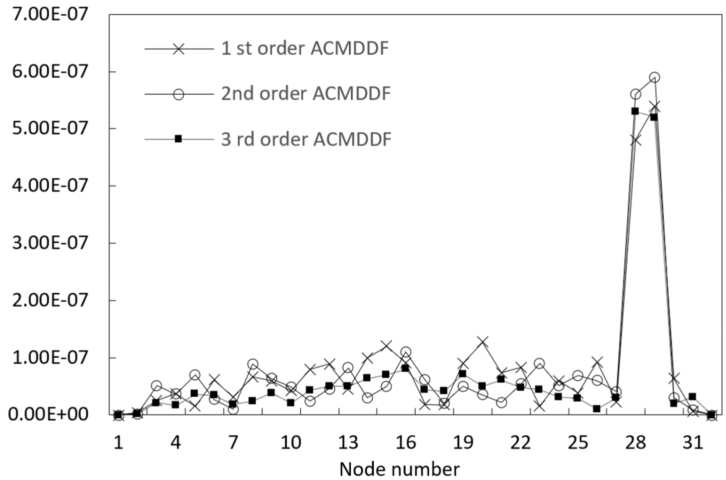


FIGURE 7. Identification results by CMD_{ave}(W2)

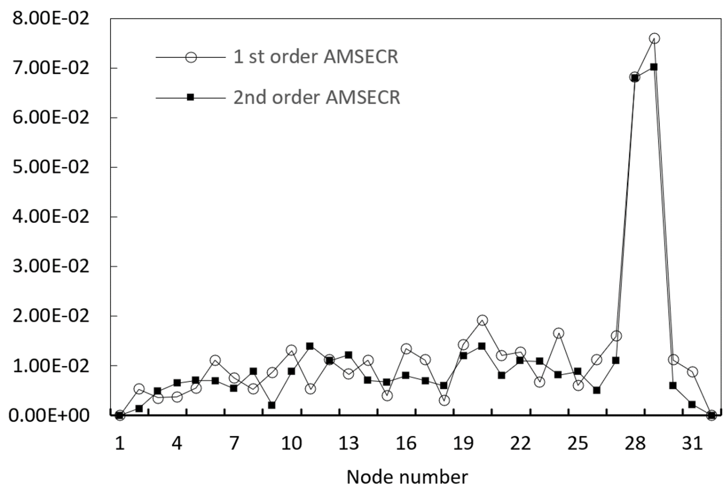


FIGURE 8. Identification results by MSE_{ave}(W2)

Fig. 4 to 12 assumes that the level of noise is 5% in single as well as multiple damage cases. The analysis of the single-damage dynamic fingerprinting identification shows that the anti-interference ability in single dynamic fingerprinting is weak and that the structural damage cannot be accurately located. The stability of the third-order CMD$_{ave}$ is obviously higher than that of its first- and second-order counterparts. The first- and second-order CMD$_{ave}$ present a discrete micro-oscillation trend and are characterized by low noise immunity. After fusing the five groups of dynamic fingerprint data using Bayes formula, the identification accuracy of the damage locations improves considerably. According to Fig. 6 and 9, significant mutations occur in the lesion area, no mutation occurs in the damage area, and identification is possible even with damage as low as 5% and 10%. Moreover, the damage locations were accurately located. Thus, unlike the case of the single index, Bayes data fusion method is very effective at avoiding misjudgments.

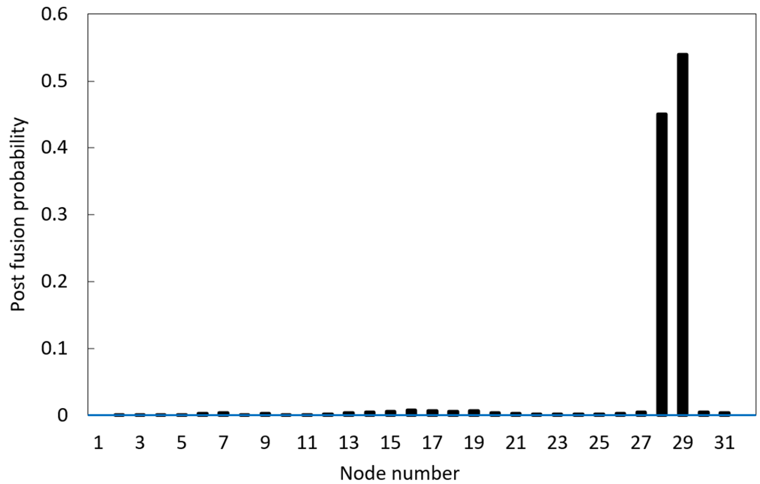


FIGURE 9. Results by the Bayes fusion method (W2)

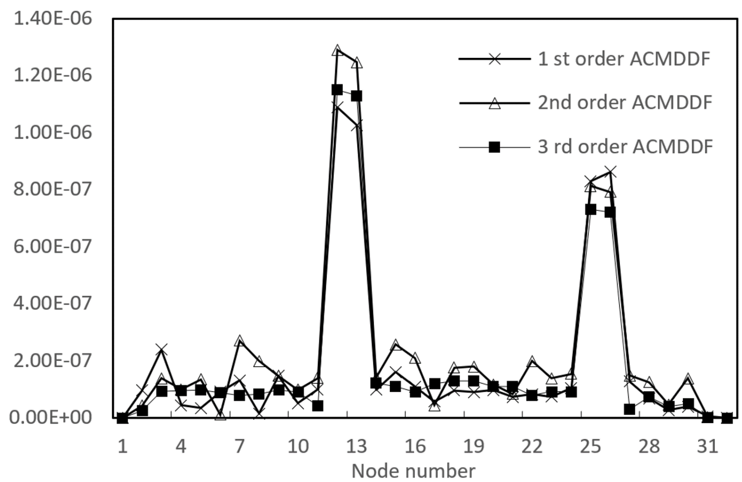


FIGURE 10. Identification results by CMD_{ave}(W3)

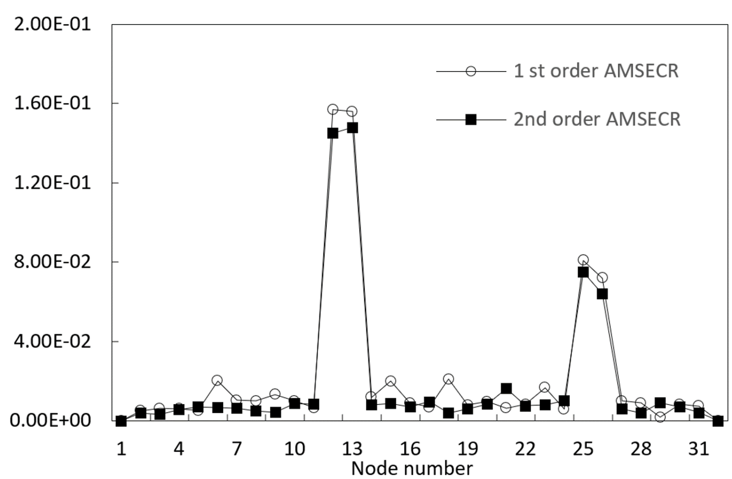


FIGURE 11. Identification results by MSE_{ave}(W3)

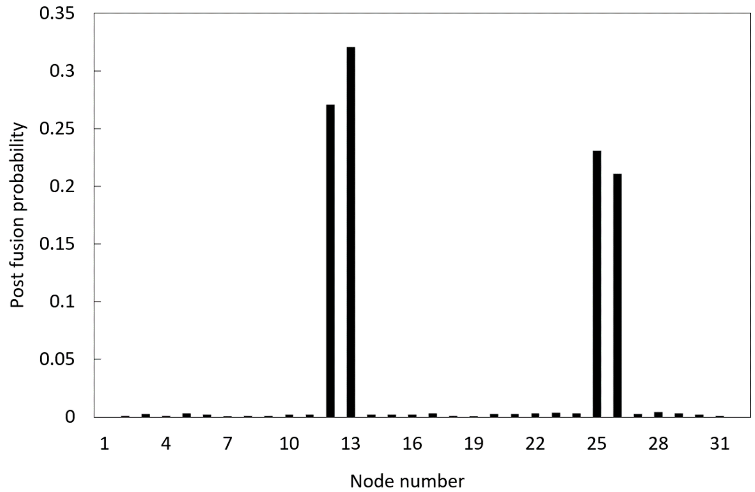


FIGURE 12. Results by the Bayes fusion method (W3)

4.4. **Degree of damage identification.** In order to verify the effectiveness of Bayes network classifier in the identification of damage degree, this study conducted an analysis assuming noise levels of 5% and 10%, and working conditions 1 and 2 as the damage conditions. Each type of damage condition produced 310 measurements, and each noise level produced 310 data test samples for these two working conditions.

The methods for adding noise and processing were the same as those for damage identification, the only difference being the manner of extracting the feature factor. If the previous damage localization confirmed that unit 4 is damaged, the extraction nodes 17 and 18 before. Thus, using the 3 order CMD_{ave} and the first two orders of MSE_{ave} would provide a total of 10 data points. The Bayes network classifier was constructed and the k-means clustering algorithm was used to analyze the discretization of the sample data [17].

The identification results pertaining to the damage degree are shown in table 2. When the ambient noise level is less than 5%, the degree of damage to the TTS can be accurately identified. When ambient noise levels rise to 10%, the degree of damage for the two conditions is misjudged slightly, but the accuracy rate still exceeds 96%. In the numerical example considered in this study, the degree of damage in the design is low and the difference between the damage categories is small. Therefore, the proposed method is promising for precise damage identification and classification for even minor damage categories in a noisy environment.

5. **Conclusion**

When there is no noise or the noise level is low, structural dynamic fingerprinting by CMD_{ave} or MSE_{ave} alone provides accurate damage identification. When the noise level is more than 5%, the recognition accuracy is slightly lower. After conducting Bayes data fusion, however, the recognition accuracy increases sharply, because the

TABLE 2. Identification of damage degree

Working condition	Noise level (%)	Accuracy of recognition (%)
W1	5	100
	10	99
W2	5	100
	10	98
W3	5	100
	10	97

identification of modal parameters can effectively eliminate the influence of testing noise. First, structural damage should be accurately located to extract its dynamic fingerprint, and then, the Bayes network classifier should be used to identify the degree of damage. The resulting recognition accuracy shows a major improvement over the results from a single index (over 96%), and the proposed method also reduces the operation time. Thus, TTS damage identification based on dynamic fingerprinting and Bayes data fusion has good fault tolerance, noise resistance, and stability.

Acknowledgments

The research described in this paper was financially supported by the Doctoral Scientific Research Foundation of Liaoning Province, supported by Liaoning science and Technology Department, project No. 20170520201.And Special fund for scientific research of Liaoning Provincial Education Department, project No. L2015222.

REFERENCES

[1] Xiufeng Wu, Hongfang Li and Ke Zhao, Study on Wind Vibration Coefficient Change Rule of Cup Type Transmission tower Considered Abrupt Change in Stiffness, J. Auto. and Control., 35(2) (2014), 182–188.

[2] Zong, Z., Niu, J, and Wang, H, A review of structural damage identification methods based on the finite element model validation, China Civil Engineering Journal, 45(8) (2012), 121–130.

[3] Chang, K.C. and Kim, C.W., Modal-parameter identification and vibration-based damage detection of a damaged steel truss bridge, Eng. Struct., 122(2016), 156–173.

[4] Khatir, A., Tehami, M., Khatir, S. and Wahab, M.A.,Multiple damage detection and localization in beam-like and complex structures using co-ordinate modal assurance criterion combined with firefly and genetic algorithms, J. Vibroeng., 18(8) (2016), 5063–5073.

[5] Roy, K., Structural damage identification using mode shape slope and curvature, J. Eng. Mech., 143(9) (2017).

[6] Sha, L. and Wang, T., Structural reliability analysis for implicit performance with Legendre orthogonal neural network method, Journal of Harbin Institute of Technology, 23(1) (2016), 60–66.

[7] Yu, J., Hu, M. and Wang, P., Evaluation and reliability analysis of network security risk factors based on D-S evidence theory, J. Intell. Fuzzy Syst., 34(2) (2018), 861–869.

[8] Riza, L.S., Janusz, A., Bergmeir, C., Chris, C., Herrera, F., Ślezak, D. and Beníteza, J.M., Implementing algorithms of rough set theory and fuzzy rough set theory in the R package ‘RoughSets, Information Sciences, 287(19) (2014), 68–89.

[9] Pandey, A.K. and Biswas, M., Damage detection in structures using changes in flexibility. Journal of Sound & Vibration, 169(1) (1994), 3–17.

[10] Sohn, H. and Law, K. H., A bayesian probabilistic approach for structure damage detection, Earthquake Engineering & Structural Dynamics, 26(12) (2015), 1259–1281.

[11] Beck, J. L., Au, S. K. and Vanik, M. W., Monitoring structural health using a probabilistic measure, Computer-aided Civil & Infrastructure Engineering, 16(1) (2010), 1–11.

[12] Yang, G.Q., Liu, Y.L. and Hong-Wen, Z., The calculation method of curvature attributes and its effect analysis, Progress in Geophysics, 5(3) (2012), 56–62.

[13] Cha, Y. and Buyukozturk, O., Structural damage detection using modal strain energy and hybrid multiobjective optimization, Comput-aided Civ. Inf. Eng., 30(5) (2015), 347–358.

[14] Wei, Z.T., Liu, J.K. and Lu, Z.R., Damage identification in plates based on the ratio of modal strain energy change and sensitivity analysis, Inverse Problems in Science & Engineering, 24(2) (2016), 265–283.

[15] Farzaneh, Y. and Tootoonchi, A.A., Global error minimization method for solving strongly nonlinear oscillator differential equations, Comput. Math. Appl., 59(8) (2010), 2887–2895.

[16] Kiviniemi, T.P. and Sauerwein, U., The effects of spatial sampling on random noise for gyrokinetic PIC simulations in real space, Comput. Phys. Commun., 203(8) (2016), 162–167.

[17] Beg, A.H., Islam, M.Z. and Estivill-Castro, V., Genetic algorithm with healthy population and multiple streams sharing information for clustering, Knowl.-Based Syst., 114(6) (2016), 61–78.